

Dynamic Soaring Revisited

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Overview

- **History**
- **DS in wind shear**
- **DS in turbulence**
- **Conclusions**
- **Q&A**

I premise that if we know anything about mechanics it is certain that a bird without working his wings cannot, either in still air or in a uniform horizontal wind, maintain his level indefinitely. For a short time such maintenance is possible at the expense of an initial relative velocity, but this must soon be exhausted. Whenever therefore a bird pursues his course for some time without working his wings, we must conclude either

- (1) That the course is not horizontal*
- (2) That the wind is not horizontal, or*
- (3) That the wind is not uniform*

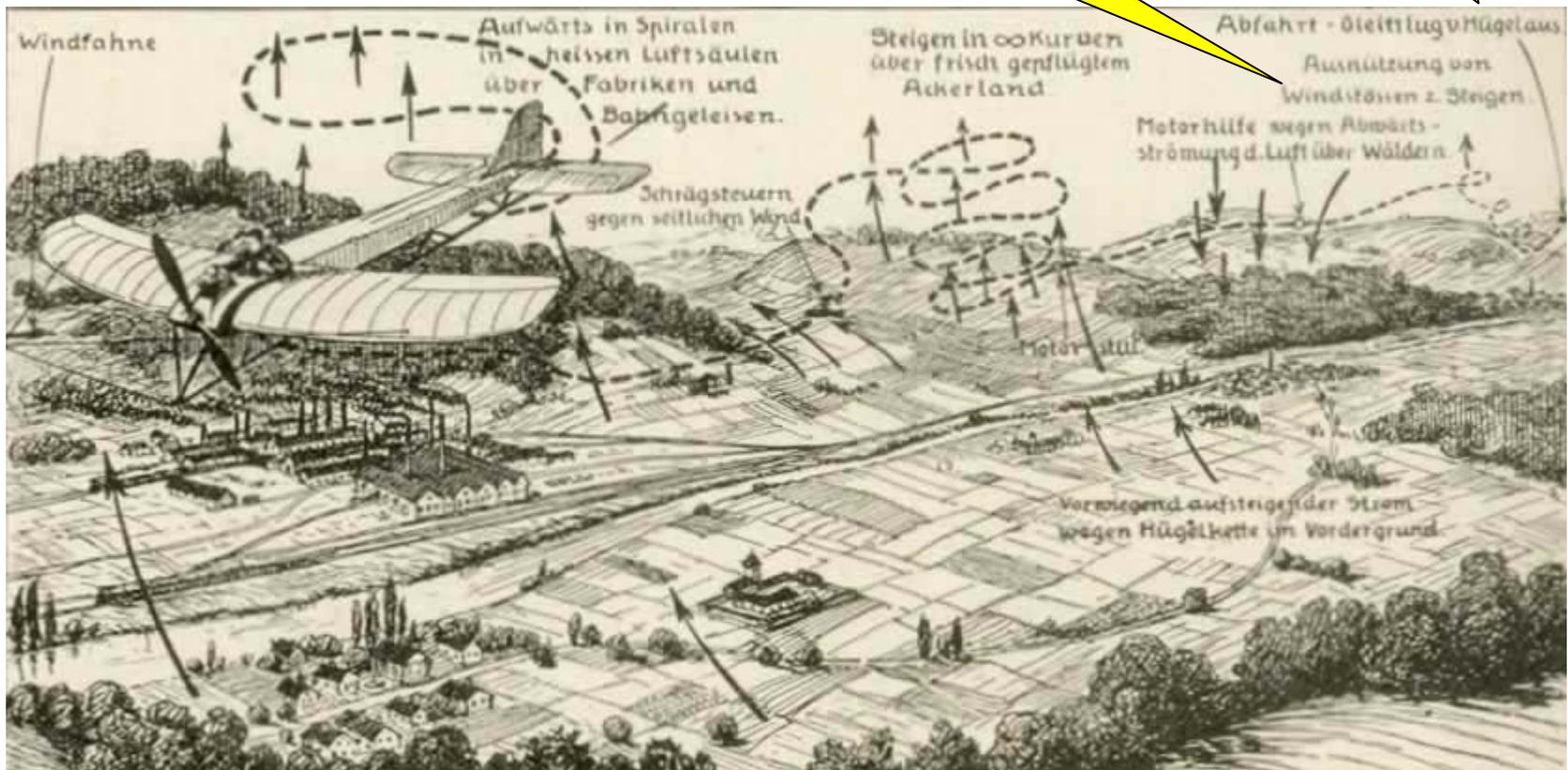
It is probable that the truth is usually represented by (1) or (2); but the question I wish to raise is whether (3) may not sometimes come into operation.

-- Lord Raleigh, Nature, 27, April 5, 1883, pp534-5

History – Wasserkuppe, Germany 1920's

Launch:
Glide from a
Hill

Use of wind
gusts to rise



Pioneers: Klemperer, Hirth, von Karman and many others; John Montgomery, Langley ("Internal Work of the Wind")

History – Idrac on Thermals

Pierre Idrac “Etudes Experimentales sur le Vol a Voile,” 1931

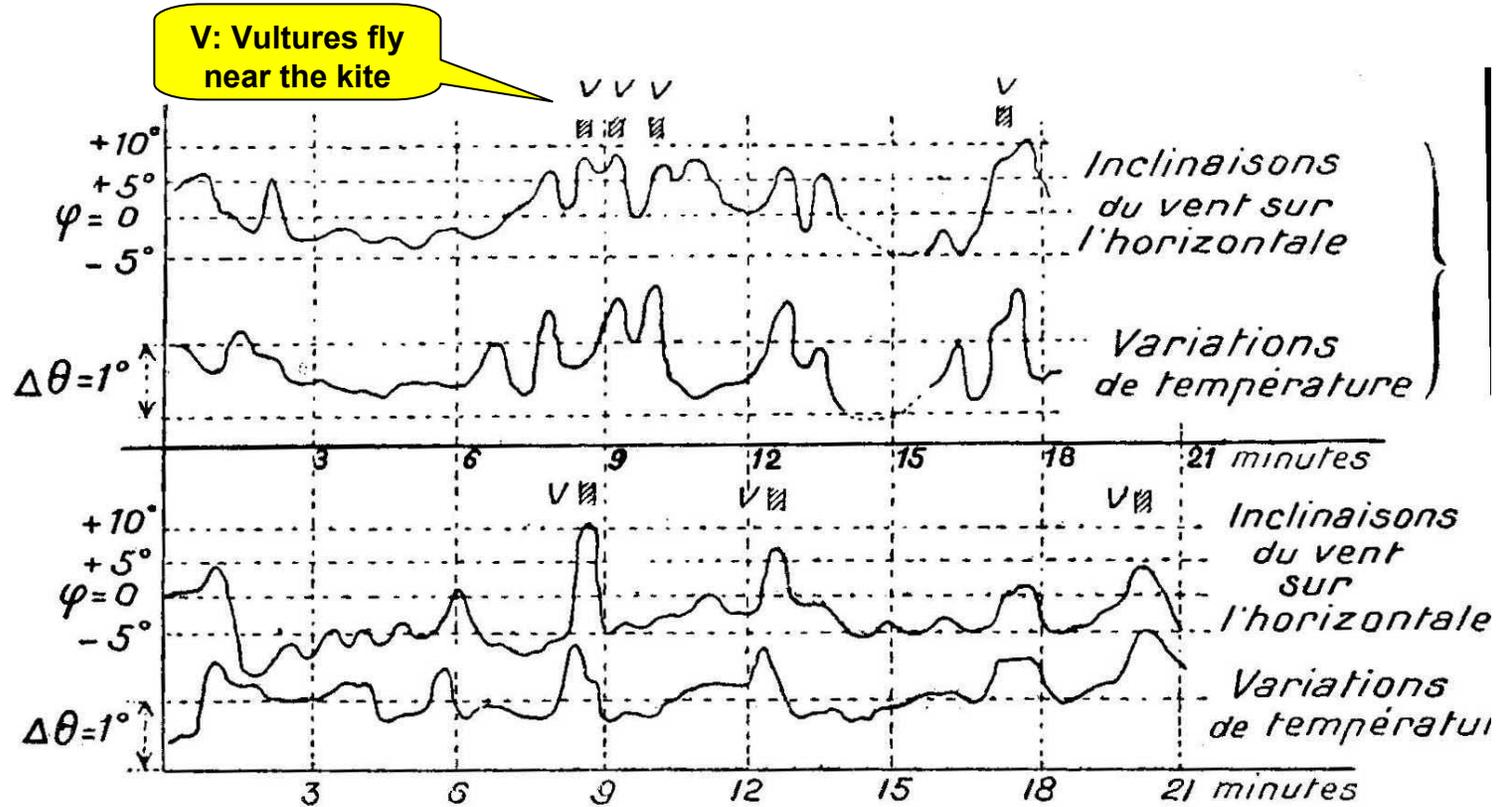
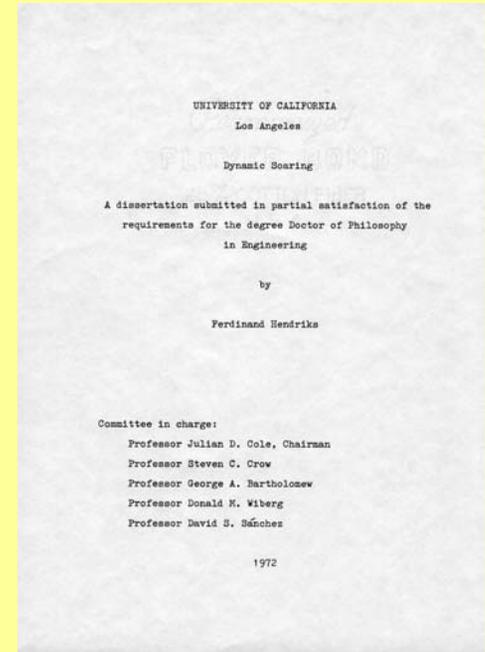
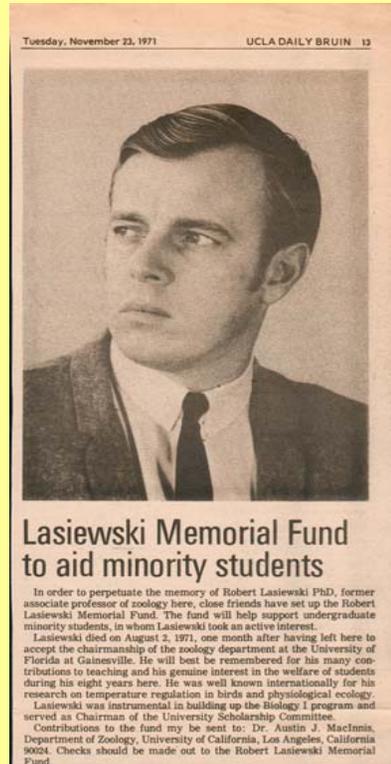


ABB. 30 u. 31. — Diagramme der Neigungswinkel des Windes gegen die Horizontale (Inclinaisons...) und der gleichzeitigen Aenderungen der Temperatur (Variations...) in Abhängigkeit von der Zeit in Minuten. — Die Schraffuren über den Kurven vv bedeuten, dass Vögel zu diesem Augenblick in unmittelbarer Nachbarschaft der Apparate fliegen.

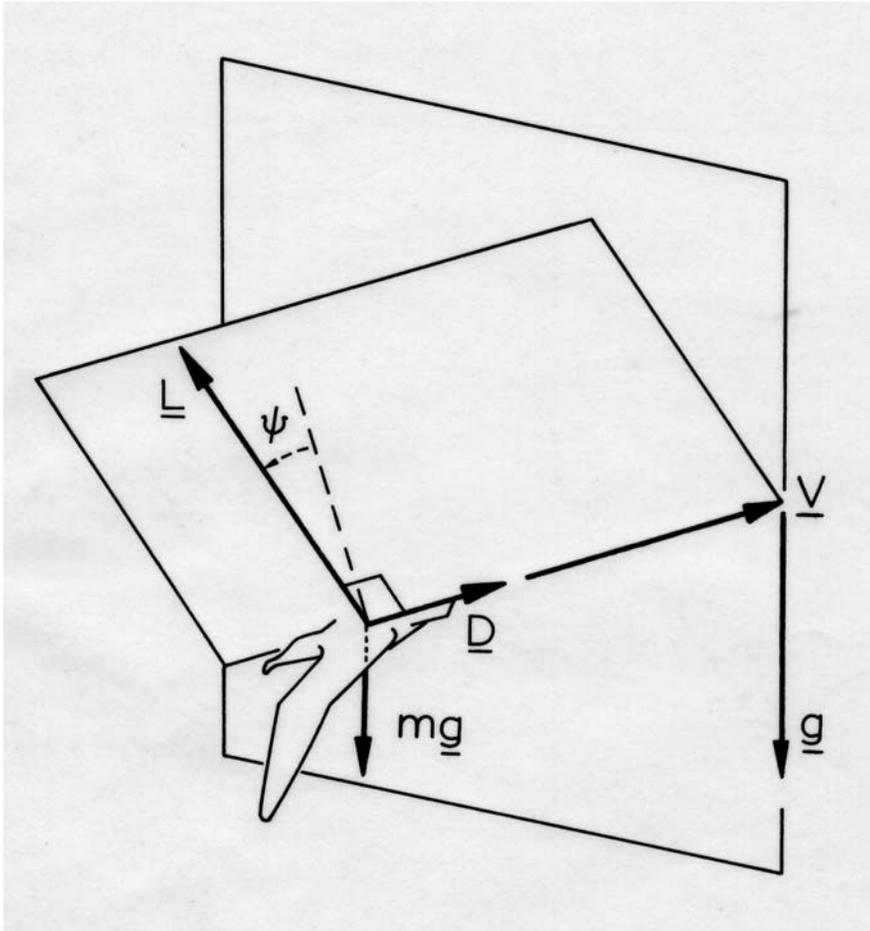
History – Applied Math/Zoology project

UCLA, 1971



Attempts to demonstrate DS with an RC glider over flat terrain ...

DS in shear – modelling the glider



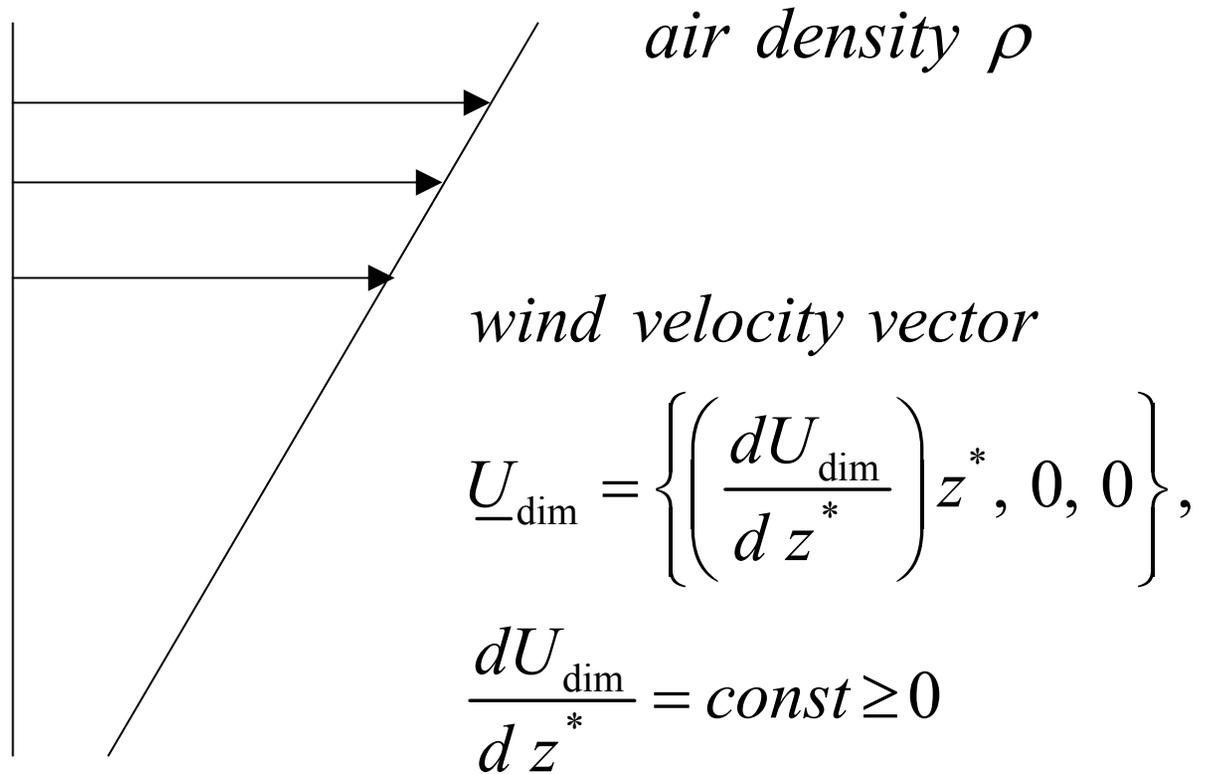
$$L = c_L \frac{1}{2} \rho V^2 S$$

$$D = c_D \frac{1}{2} \rho V^2 S$$

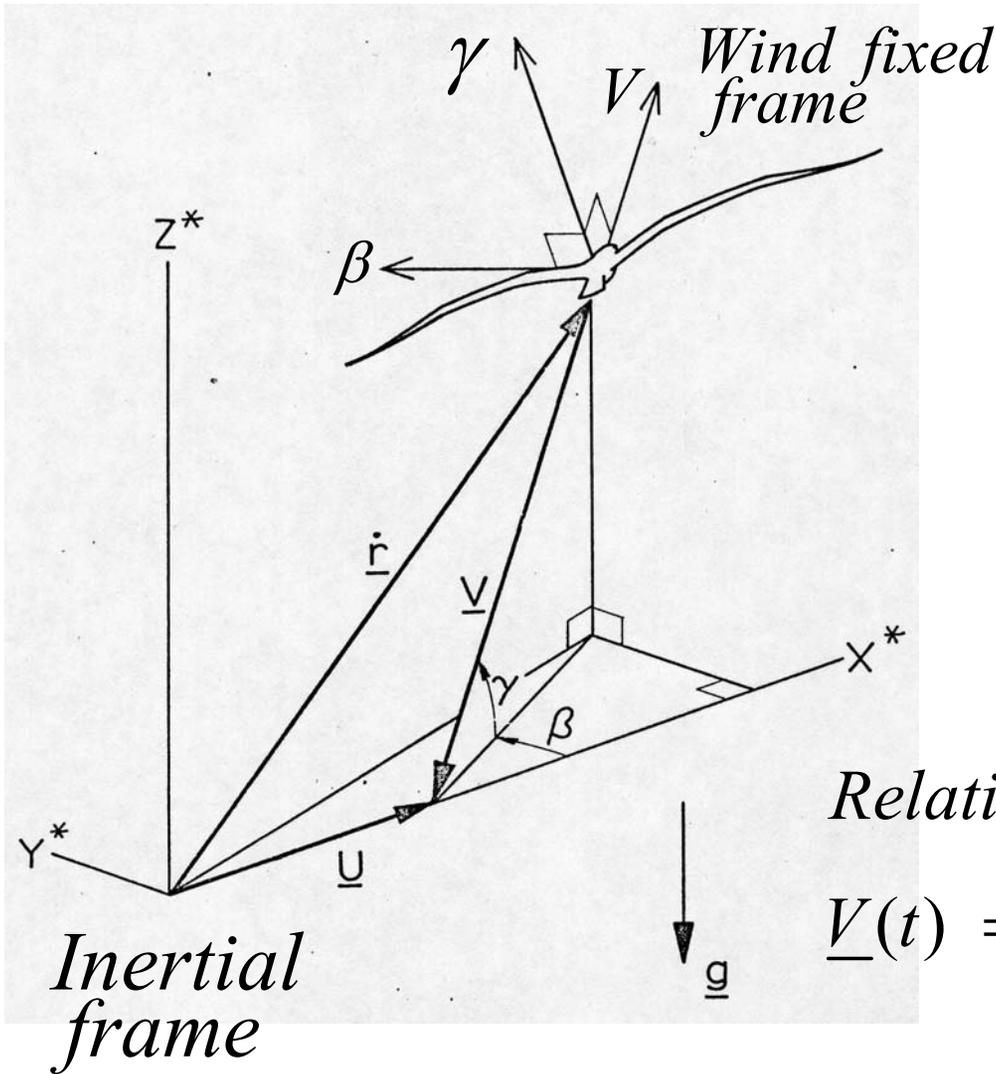
$$c_D = c_{D_0} + k c_L^2$$

*control $\psi(t)$, $\alpha(t)$
mass m , wing area S*

DS in shear – modelling the wind



DS in shear – “Wind-fixed” axes



Relative (apparent) wind :

$$\underline{V}(t) = \left\{ \underline{U}(\underline{x}^*) \right\}_{\underline{x}^* = \underline{r}(t)} - \dot{\underline{r}}(t)$$

DS in shear – inertial eqns of motion

Inertial frame:

$$m \underline{\ddot{r}}(t) = \underline{L} + \underline{D} + m \underline{g}$$

$$l_c = 2m / \rho S \quad \text{characteristic speed}$$

$$V_c = \sqrt{g l_c} \quad \text{characteristic speed}$$

$$t_c = \sqrt{\frac{l_c}{g}} \quad \text{characteristic time}$$

DS in shear – “wind-fixed” eqns of motion; linear shear

wind fixed equations

made non-dimensional using V_c and t_c

$$\dot{V} = -V \frac{dU}{dz} \boxed{\sin \gamma \cos \gamma \cos \beta} - c_D V^2 - \sin \gamma$$

DS rule

drag

gravity

$$V \dot{\gamma} = V \frac{dU}{dz} \sin^2 \gamma \cos \beta + c_L V^2 \cos \psi - \cos \gamma$$

$$(V \cos \gamma) \dot{\beta} = V \frac{dU}{dz} \sin \gamma \sin \beta + c_L V^2 \sin \psi$$

$$\dot{x} = V \cos \gamma \cos \beta + U$$

$$\dot{y} = V \cos \gamma \sin \beta$$

$$\dot{z} = V \sin \gamma$$

DS in shear – Energy eqn

Indicated air speed Altitude

$$\frac{d}{dt} \left(V^2 / 2 + z \right) = -V^2 \frac{dU}{dz} \sin 2\gamma \cos \beta - c_D V^3$$

Rate of change of total relative energy Rate of relative Energy gain or loss Rate of dissipation

DS in shear – Necessary Cond'n to stay aloft

From the energy equation, we want the rate of change of the total energy to be greater or equal to zero, thus

$$-V^2 \frac{dU}{dz} \sin 2\gamma \cos \beta - c_D V^3 \geq 0$$

$$\frac{dU}{dz} \geq c_D V$$

In dimensional terms:

$$\left(\frac{dU}{dz} \right)_{\text{dim}} \geq g \sqrt{\left(\frac{c_D^2}{c_L} \right) \left(\frac{2\rho}{W/S} \right)}$$

Wandering albatross:

$$W/S = 150 \text{ N/m}^2$$

$$c_L = 1 \quad c_D = .1$$

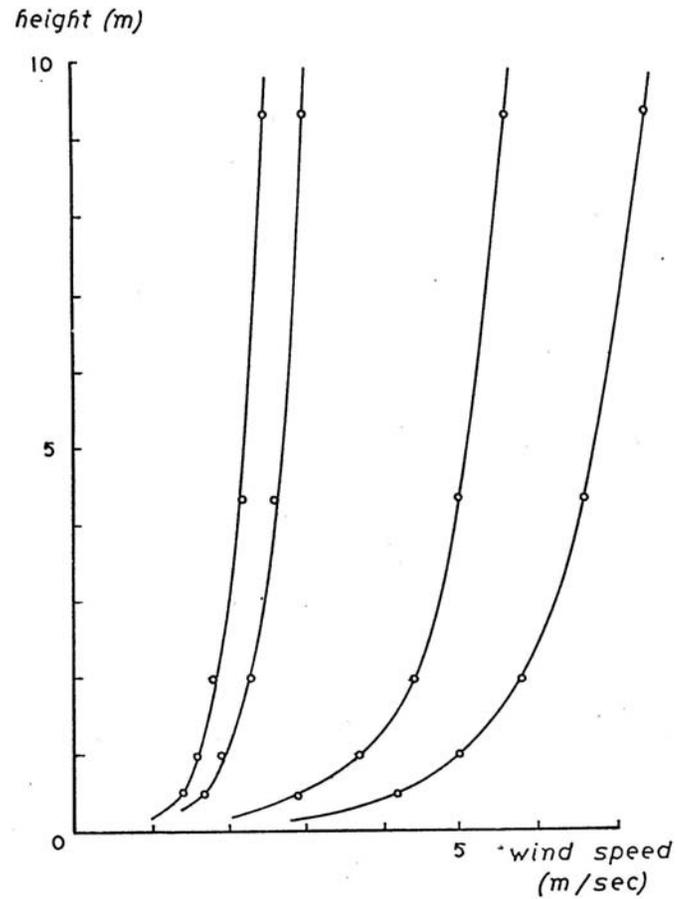
$$g = 9.81 \text{ m/sec}^2$$

$$\rho = 1.25 \text{ kg/m}^3$$

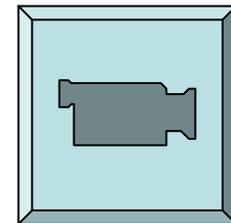
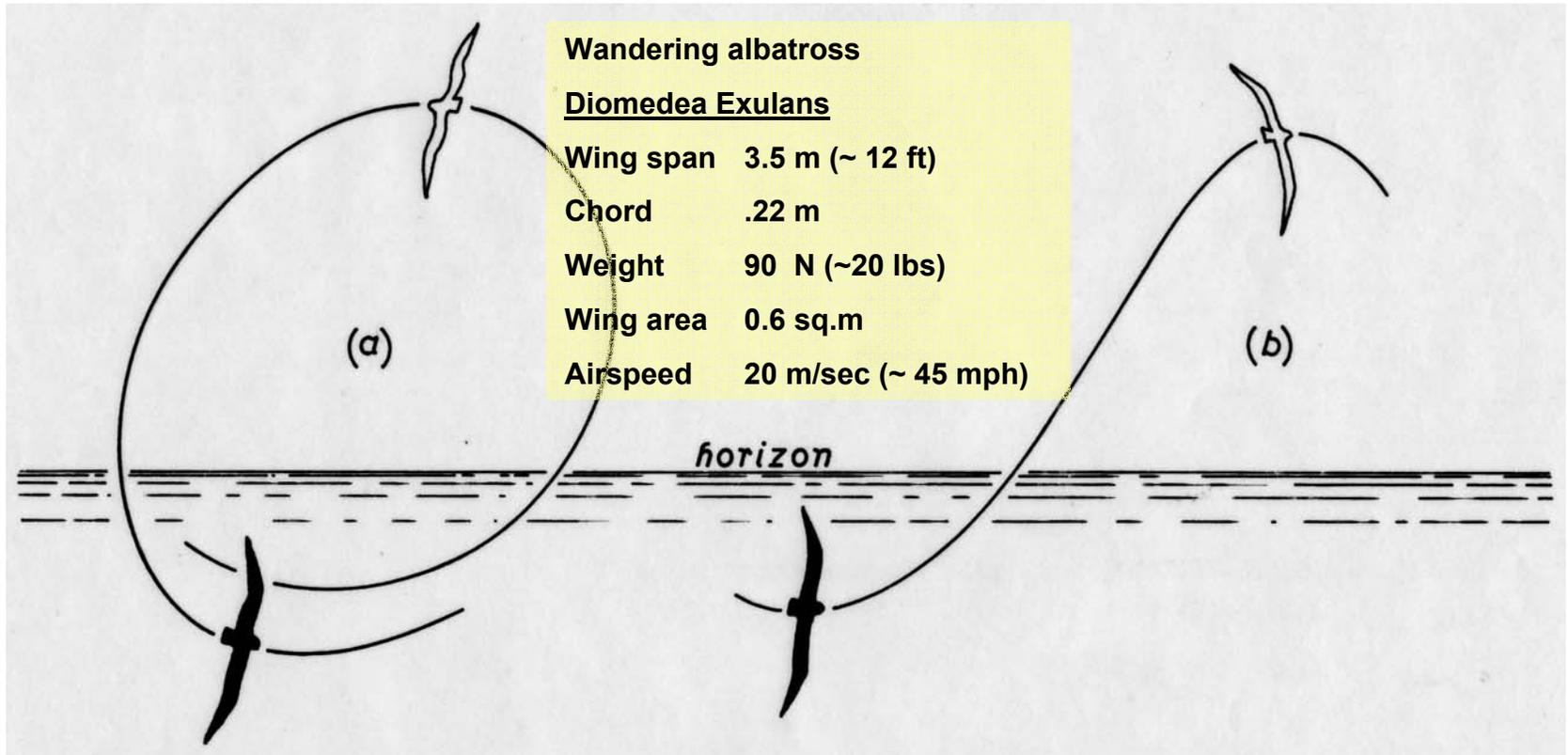
$$(dU/dz) \geq .108 \text{ sec}^{-1}$$

High wing loading and laminar flow airfoils are favored.
Vertical wind shears exceeding .1 sec⁻¹ are common.

DS in shear – wind profiles over land

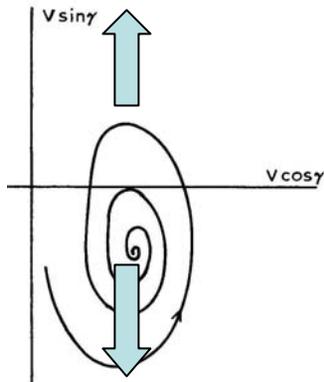


DS in shear – trajectories observed by Idrac

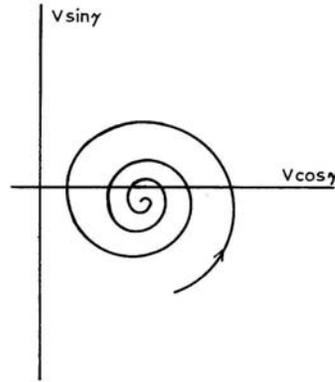


DS in shear – phaseplane portraits

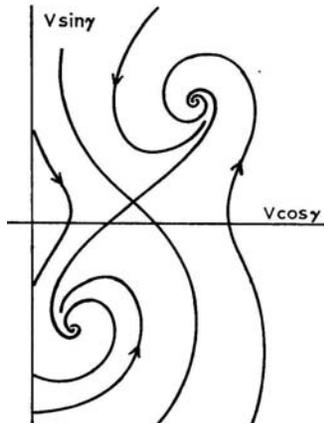
plots of the relative wind vector V



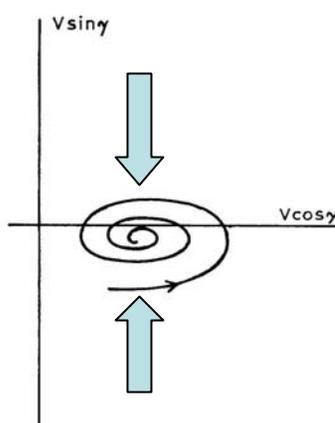
$\beta = \pi$, $\frac{dU}{dz}$ small.



$\beta = 0$, $\frac{dU}{dz}$ small.



$\beta = \pi$, $\frac{dU}{dz}$ large.

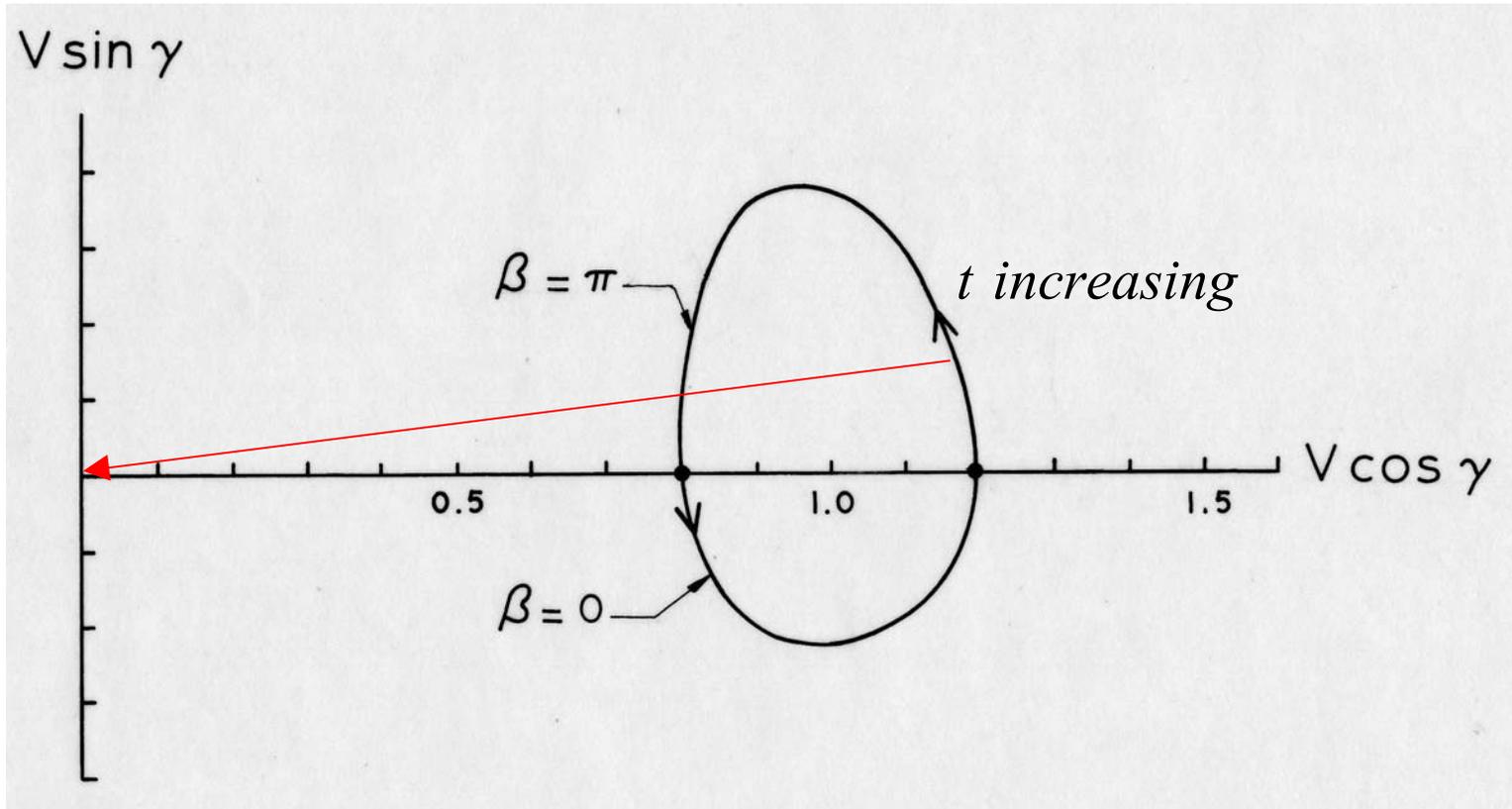


$\beta = 0$, $\frac{dU}{dz}$ large.

$\beta = 0$ with the wind
 $\beta = \pi$ into the wind

The phugoid oscillation
Is “pulled” by the wind
shear.

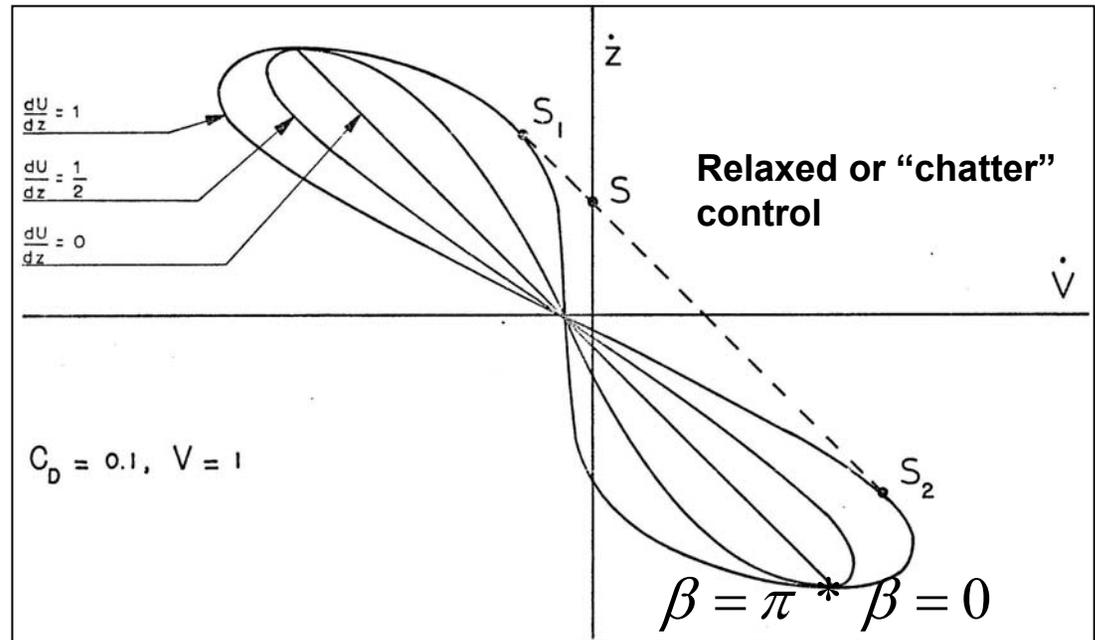
DS in shear – piecing phaseplanes together, drag = 0



DS in shear – Contensou’s approach

Assumptions:

- $\beta = 0$ or π (+, - sign)
- eliminate γ
- $V = \text{const}$



$$\dot{V} = -c_D V^2 \mp \frac{dU}{dz} \dot{z} \sqrt{1 - \left(\frac{\dot{z}}{V}\right)^2} - \frac{\dot{z}}{V}$$

Recall first
Eqn. of motion

DS in shear – Perturbation approach

$$\psi = \text{const}, \quad \frac{dU}{dz} = O(1)$$

$$V(t) = V_0(t) + \varepsilon V_1(t) + \dots$$

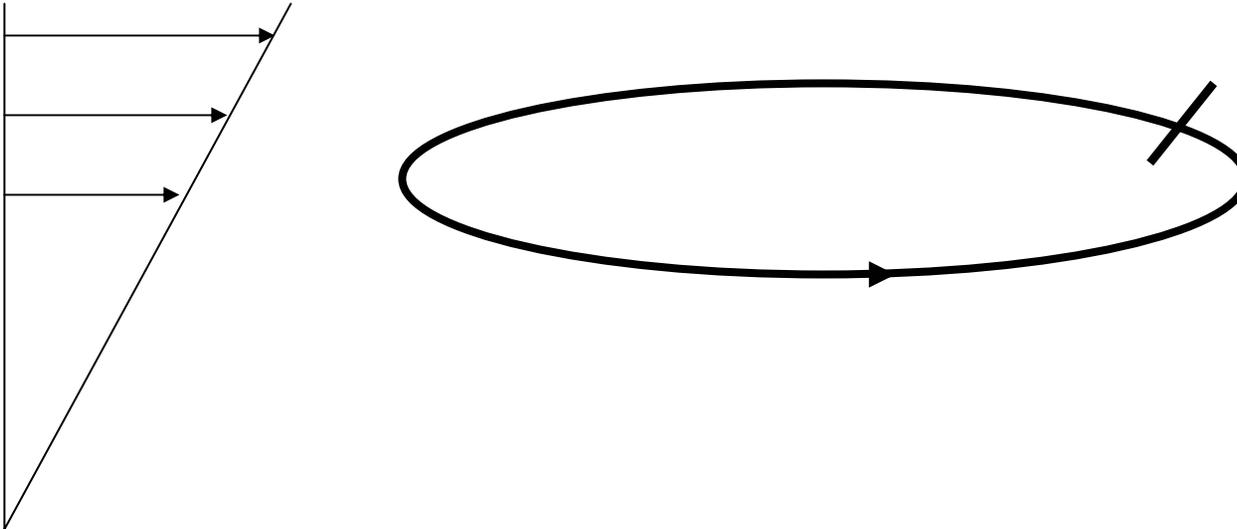
$$\gamma(t) = \varepsilon \gamma_1(t) + \dots$$

$$\beta(t) = \beta_0(t) + \varepsilon \beta_1(t) + \dots$$

$$c_L = c_{L_0}(t) + \varepsilon c_{L_1}(t) + \dots$$

$$c_D = \varepsilon c_D$$

DS in shear – $O(1)$ motion (zero drag)



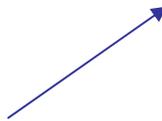
To first order, the motion is a horizontal circle, flown at constant speed V_0
Corresponding to some bank angle ψ

DS in shear – Mathieu eqn.

At $O(\varepsilon)$ we obtain

$$\ddot{\gamma}_1 + \left(\frac{2}{V_0^2} + \frac{2}{V_0} \frac{dU}{dz} \cos \omega_0 t \right) \gamma_1 = -2c_D + \frac{\dot{c}_{L_1}}{c_{L_0} V_0}$$

“pumping term” 

control 

Seek a non-sinking solution despite the term $-2c_D$ on r.h.s.

DS in shear – Bank rule

The control $c_{L_1}(t)$ that achieves the goal is

$$c_{L_1}(t) = -\frac{1}{2}\sqrt{6} c_D \sin 2\omega_0(t)$$

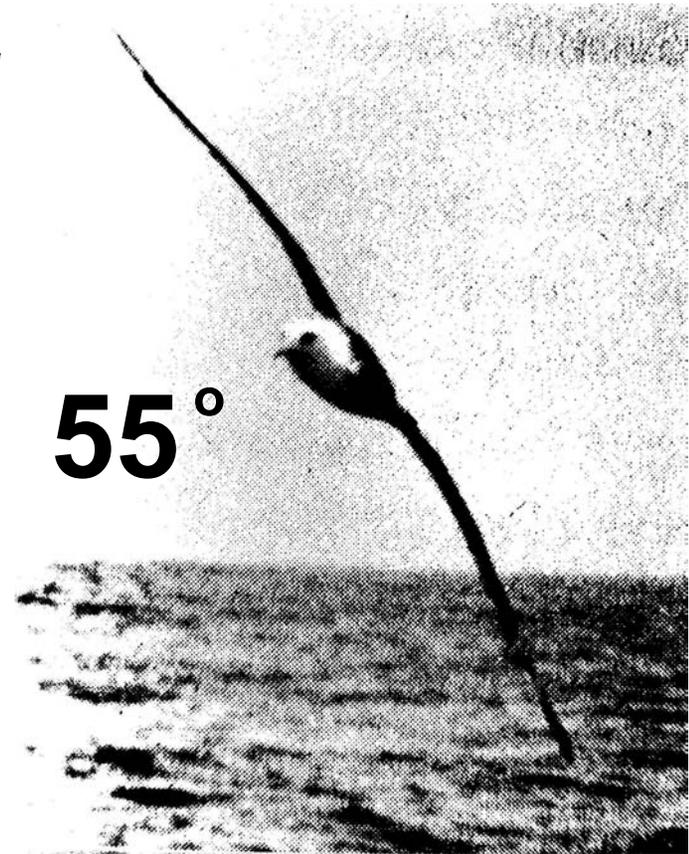
provided the angle of bank ψ satisfies

$$\operatorname{tg} \psi = \sqrt{2}$$

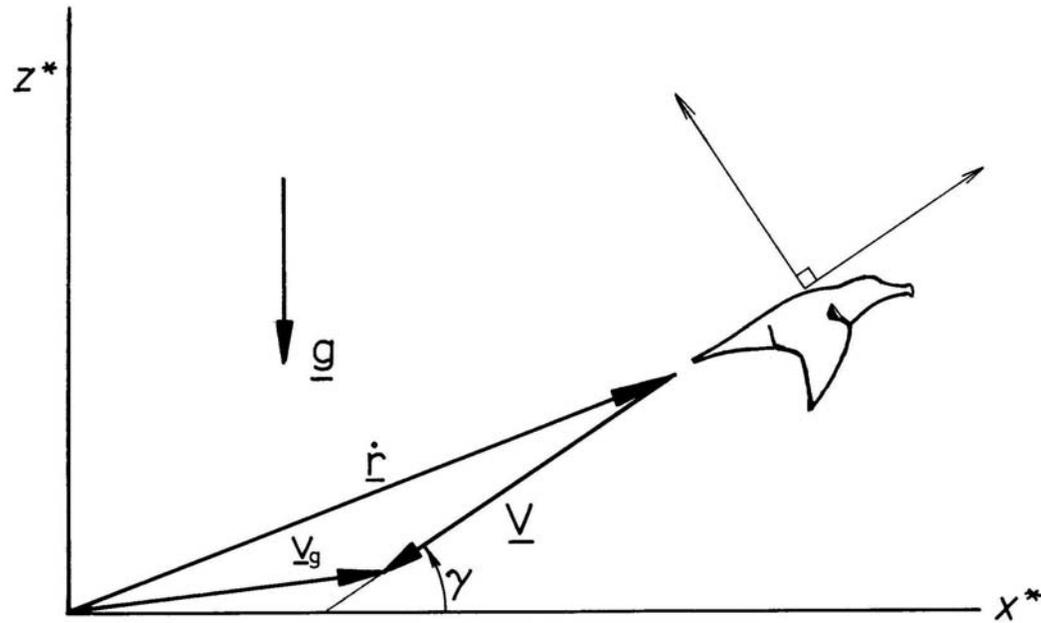
or

$$\psi = 55^\circ$$

This agrees with Idrac's observations!



DS in GUSTS, 2D



$$\underline{V}(t) = \underline{v}_g(t) \left\{ x^* \right\}_{\underline{x}^* = \underline{r}(t)} - \dot{\underline{r}}(t)$$

DS in gusts – Non-dim. eqns of motion in wind-fixed frame

$$\dot{V} = -V \left\{ u_{g_x} \cos^2 \gamma + (u_{g_z} + w_{g_x}) \sin \gamma \cos \gamma + w_{g_z} \sin^2 \gamma \right\} + \text{gust/glider}$$

$$- \left\{ (u_g w_{g_x} + w_g w_{g_z}) \sin \gamma + (u_g u_{g_x} + w_g u_{g_z}) \cos \gamma \right\} + \text{gust/gust}$$

$$- c_D V^2 - \sin \gamma \quad \text{gravity}$$

drag

$$V \dot{\gamma} = V \left\{ u_{g_z} \sin^2 \gamma + (u_{g_x} - w_{g_z}) \sin \gamma \cos \gamma - w_{g_x} \cos^2 \gamma \right\} + \text{gust/glider}$$

$$+ \left\{ (u_g u_{g_x} + w_g u_{g_z}) \sin \gamma - (u_g w_{g_x} + w_g w_{g_z}) \cos \gamma \right\} + \text{gust/gust}$$

$$+ c_L V^2 - \cos \gamma \quad \text{gravity}$$

lift

$$\dot{x} = u_g + V \cos \gamma$$

$$\dot{z} = w_g + V \sin \gamma$$

DS in gusts – Solution outline

Expand in terms of $\varepsilon = (v_g)_{rms} / V_c$ (weak gusts)

$$V = V_0 + \varepsilon V_1 + \dots$$

$$\gamma = \varepsilon \gamma_1 + \dots$$

$$u_g = \varepsilon u$$

$$w_g = \varepsilon w$$

$$c_L = c_{L_0} + \varepsilon c_{L_1} + \dots \leftarrow \text{control}$$

$$c_D = \varepsilon^2 c_{D_2}$$

Turbulent energy has the Kolmogorov form

$$E(k) = \bar{\alpha} \varepsilon^{2/3} k^{-5/3}$$

$$\varepsilon = \frac{\mathcal{E}_{dim}}{l_{char}^{1/2} g^{3/2}}$$

Gusts appear more energetic to
Gliders with small char. length (short, fast phugoid)

Turbulent dissipation rate

Turbulence is "frozen" and gusts are sampled along a straight line

DS in gusts – control

Try the feedback control

$$c_{L_1} = AV_1 - B\gamma_1$$

then, after eliminating V_1 and using $d/dt = V_0 d/dx$

$$\gamma_{1,xx} + B\gamma_{1,x} + K^2\gamma_1 = -\frac{w_{xx}}{V_0} - K^2 V_0 u_x \quad (B \text{ acts as artificial damping})$$

where

$$K = \frac{1}{V_0} \left\{ 2c_{L_0} + AV_0 \right\}^{1/2} \quad (\text{n.d. waveno. of phugoid})$$

Solve for γ_1 using Fourier transforms and use it in the $O(\varepsilon^2)$ eqns
after much reduction we find the turbulence / drag balance
satisfied for sustained horizontal flight:

$$B \int_{-\infty}^{\infty} \frac{k^4 W(k) dk}{(K^2 - k^2) + B^2 k^2} + BK^2 \int_{-\infty}^{\infty} \frac{k^2 U(k) dk}{(K^2 - k^2) + B^2 k^2} - c_{D_2} V_0^2 = 0$$

DS rule:
Maximize
artificial
damping of
the phugoid

DS in gusts – final turbulence/drag balance

U(k) and W(k) are the longitudinal and lateral turbulence spectra

Both can be expressed in terms of the turbulent dissipation rate \mathcal{E}

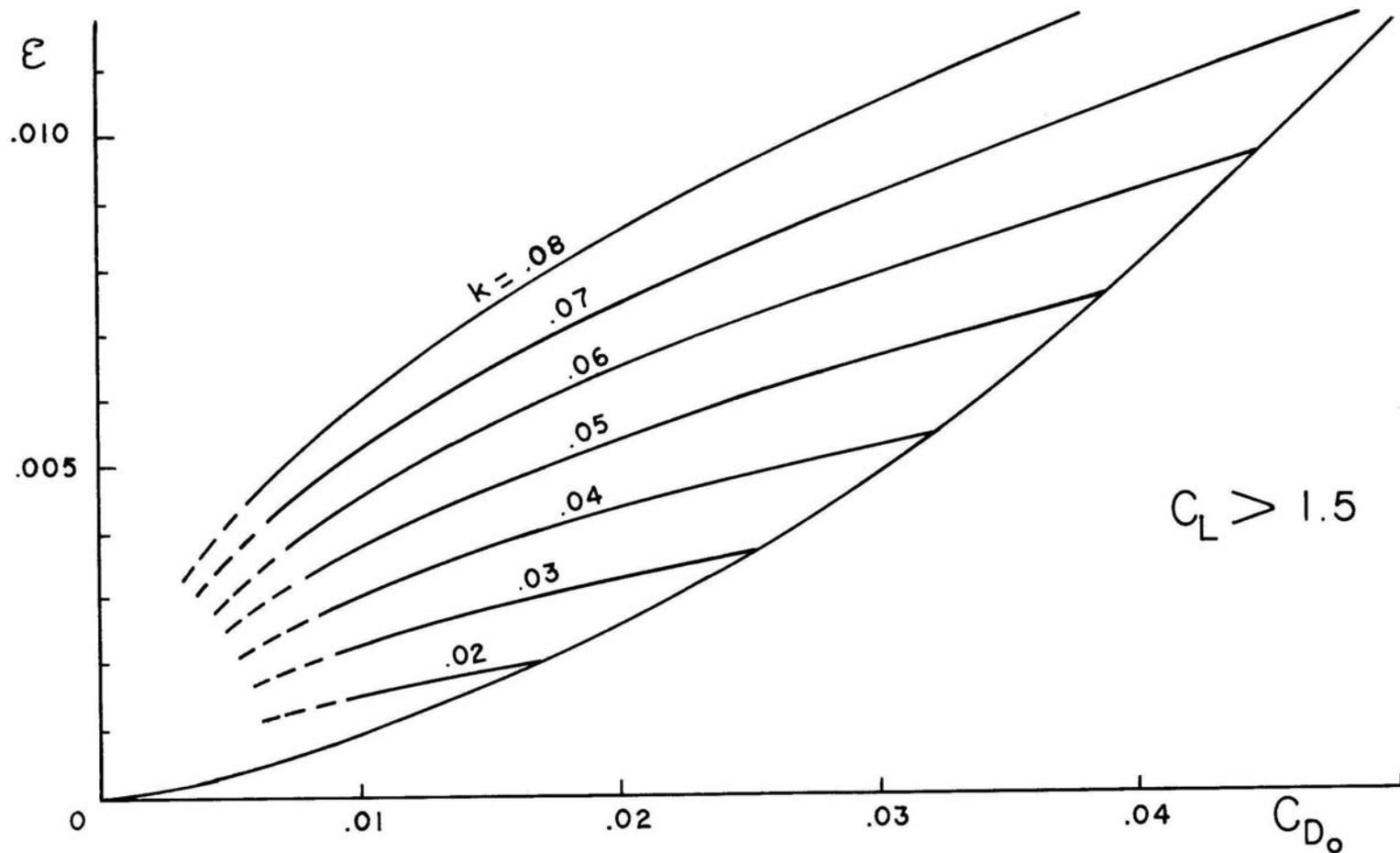
For example, after (rather arbitrarily) choosing $B = 2K$ we get

$$\frac{4}{15} \pi \sqrt{3} \bar{\alpha} \mathcal{E}^{2/3} K^{1/3} - c_{D_2} V_0^2 = 0$$

Using realistic values that approximate a Frigate bird

flying at 30m altitude in turbulence we require $\mathcal{E} = .795 \text{ m}^2 / \text{sec}^3$

DS in gusts – Required turbulent dissipation for given aerodynamic properties



Conclusions

- **DS in shear is feasible for RC gliders and birds in naturally occurring wind gradients. High glide speeds are required. Theoretical predictions are being confirmed but not under controlled conditions.**
- **A figure of merit was derived for shear-soaring gliders**
- **A simple strategy for DS in (linear) shear is to synchronize the turning motion with the phugoid oscillation. This happens at a bank angle of 55 degrees.**
- **DS in turbulence requires low speed. Theoretical analyses with fewer restrictions must be done. Artificial Damping of the phugoid motion appears to be a promising control strategy.**